Bohr Model of the Atom

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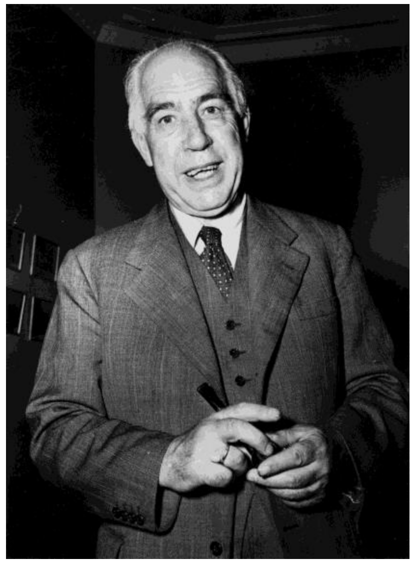
Abstract

Through his experiments, the physicist Neils Bohr improved upon Rutherford's model of the atom and developed his model of the atomic structure in 1913 that succeeded in explaining the spectral features of the hydrogen atom. A simple means for extending the conventional non-relativistic Bohr model of the atom to include the wave nature of electrons is presented. As the derivation requires basic knowledge of classical and wave mechanics, it can be taught in standard courses in modern physics and introductory quantum mechanics.



Niels Bohr was a Danish physicist who is generally regarded as one of the foremost physicists of the 20th century. He was the first to apply the quantum concept, which restricts the energy of a system to certain discrete values, to the problem of atomic and molecular structure. For that work he received the Nobel Prize for Physics in 1922. His manifold roles in the origins and development of quantum physics may be his most-important contribution, but through his long career his involvements were substantially broader, both inside and outside the world of physics.

In 1911, fresh from completion of his PhD, the young Danish physicist **Niels Bohr** left Denmark on a foreign scholarship headed for the Cavendish Laboratory in Cambridge to work under J. J. Thomson on the structure of atomic systems. At the time, Bohr began to put forth the idea that since light could no long be treated as continuously propagating waves, but instead as **discrete energy packets** (as articulated by Planck and Einstein), why should the classical Newtonian mechanics on which **Thomson's model** was based hold true? It seemed to Bohr that the **atomic model** should be modified in a similar way. If electromagnetic energy is quantized, i.e. restricted

to take on only integer values of $\mathbf{h\upsilon}$, where υ is the frequency of light, then it seemed reasonable that the mechanical energy associated with the energy of atomic electrons is also quantized.

However, Bohr's still somewhat vague ideas were not well received by **Thomson**, and Bohr decided to move from Cambridge after his first year to a place where his concepts about **quantization of electronic motion** in atoms would meet less opposition. He chose the University of Manchester, where the chair of physics was held by Ernest Rutherford. While in Manchester, Bohr learned about the nuclear model of the atom proposed by Rutherford. To overcome the difficulty associated with the classical collapse of the electron into the nucleus, Bohr proposed that the orbiting electron could only exist in certain special states of motion - called **stationary states**, in which no electromagnetic radiation was emitted. In these states, the angular momentum of the electron **L** takes on integer values of Planck's constant divided by $\boldsymbol{2\pi}$, denoted by $\boldsymbol{\hbar} = \dfrac{\mathbf{h}}{\mathbf{2\pi}}$ (pronounced h-bar). In these stationary states, the electron angular momentum can take on values $\boldsymbol{\hbar}$, $\boldsymbol{2\hbar}$, $\boldsymbol{3\hbar}$... but never non-integer values. This is known as **quantization of angular momentum**, and was one of Bohr's key hypotheses.

He imagined the atom as consisting of electron waves of wavelength $\boldsymbol{\lambda} = \dfrac{\mathbf{h}}{\mathbf{mv}} = \dfrac{\mathbf{h}}{\mathbf{p}}$ endlessly circling atomic nuclei. In his picture, only orbits with circumferences corresponding to an integral multiple of electron wavelengths could survive without **destructive interference** (i.e., $\mathbf{r} = \dfrac{\mathbf{n}\hbar}{\mathbf{mv}}$ could survive without destructive interference). For circular orbits, the position vector of the electron **r** is always perpendicular to its linear momentum p. The angular momentum **L** has magnitude **mvr** in this case. Thus Bohr's postulate of quantized angular momentum is equivalent to $\mathbf{mvr} = n\hbar$ where **n** is a positive integer called principal quantum number. It tells us what energy level the electron occupies.

Since $\lambda = \dfrac{\mathbf{h}}{\mathbf{mv}} = \dfrac{\mathbf{h}}{\mathbf{p}}$ (de Broglie relation),

$$p\, v_p = \frac{\mathbf{hv_p}}{\boldsymbol{\lambda}} = h\upsilon = \hbar\omega$$

where $\hbar = \frac{\mathbf{h}}{\mathbf{2\pi}}$ is the reduced Planck constant, ω is the angular frequency, and v_p is the phase velocity.

$$p\ v_p = \frac{\hbar \mathbf{v}}{\mathbf{r}}$$

Since $n\hbar = pr$ (quantization of angular momentum),

$$\boxed{v = n\ v_p}$$

The velocity of the electron or the group velocity of the corresponding matter wave associated with the electron is the integral multiple of the **phase velocity** of the corresponding matter wave associated with the electron.

Since n is a positive integer > 0,

$$v \geq v_p$$

By the de Broglie hypothesis, we see that

$$\frac{\mathbf{pv_p}}{\boldsymbol{\lambda}} = \frac{\mathbf{h\upsilon}}{\boldsymbol{\lambda}}$$

$$\frac{\mathbf{pv}}{\mathbf{n}\boldsymbol{\lambda}} = \frac{\mathbf{h\upsilon}}{\boldsymbol{\lambda}}$$

Substituting $n\lambda = 2\pi r$,

$$\frac{\mathbf{mv^2}}{\mathbf{r}} = 2\pi \frac{\mathbf{h\upsilon}}{\boldsymbol{\lambda}}$$

The classical description of the nuclear atom is based upon the Coulomb attraction between the positively charged nucleus and the negative electrons orbiting the nucleus. Furthermore, we consider only circular orbits. The electron, with mass **m** and charge $\boldsymbol{e}^-$ moves in a circular orbit

of radius **r** with constant velocity **v**. The attractive **Coulomb force** provides the necessary acceleration to maintain orbital motion. (Note we neglect the motion of the nucleus since its mass is much greater than the electron.) The total force on the electron is thus

$$F = \frac{\mathbf{Ze^2}}{\mathbf{4\pi\varepsilon_0 r^2}} = \frac{\mathbf{mv^2}}{\mathbf{r}}$$

where $\varepsilon_0 = 8.854 \times 10^{-12} \frac{F}{m}$ is the permittivity of free space.

$$\boxed{F = 2\pi \frac{\mathbf{h\upsilon}}{\boldsymbol{\lambda}} \quad \text{i.e., } F > \frac{\mathbf{h\upsilon}}{\boldsymbol{\lambda}}}$$

$$-\frac{\mathbf{Ze^2}}{\mathbf{4\pi\varepsilon_0 r}} = -\,2\pi r\,\frac{\mathbf{h\upsilon}}{\boldsymbol{\lambda}}$$

Substituting $2\pi r = n\lambda$,

$$-\frac{\mathbf{Ze^2}}{\mathbf{4\pi\varepsilon_0 r}} = -\,n\,h\upsilon$$

The potential energy of the electron is just given by the Coulomb potential:

$$U = -\frac{\mathbf{Ze^2}}{\mathbf{4\pi\varepsilon_0 r}} = -\,n\,h\upsilon$$

$$\boxed{U = -\,n\,h\upsilon}$$

The potential energy of the electron is the integral multiple of $-\,h\upsilon$. The negative sign indicates that it takes energy to pull the orbiting electron away from the nucleus.

Since n is a positive integer > 0,

$$U \geq h\upsilon$$

From the equation:

$$K = \frac{1}{2} mv^2$$

we can determine the kinetic energy of the electron (neglecting relativistic effects)

$$K = \frac{pv}{2}$$

Substituting $p = \frac{n\hbar}{r}$,

$$K = \frac{n\hbar v}{2r} = \frac{n\hbar\omega}{2}$$

$$\boxed{K = \frac{nh\upsilon}{2}}$$

The kinetic energy of the electron is the integral multiple of $\frac{h\upsilon}{2}$.

Since n is a positive integer > 0,

$$K \geq \frac{h\upsilon}{2}$$

The total energy E = K + U is thus

$$E = \frac{nh\upsilon}{2} + (-nh\upsilon)$$

$$\boxed{E = -\frac{nh\upsilon}{2}}$$

The total energy of the electron is the integral multiple of $-\frac{\mathbf{h\upsilon}}{\mathbf{2}}$. The frequency of radiation absorbed or emitted when transition occurs between two stationary states that differ in energy by ΔE, is given by:

$$\upsilon_{photon} = \frac{\Delta E}{h} = \frac{E_2 - E_1}{h}$$

where E_1 and E_2 are the energies of the lower and higher allowed energy states respectively. This expression is commonly known as **Bohr's frequency rule**.

$$\upsilon_{photon} = \frac{-\frac{n_2 h\upsilon_2}{2} - (-\frac{n_1 h\upsilon_1}{2})}{h}$$

$$\boxed{n_1\upsilon_1 - n_2\upsilon_2 = 2\upsilon_{photon}}$$

In physics (specifically, celestial mechanics), escape velocity is the minimum speed needed for an electron to escape from the electrostatic influence of a nucleus. If the kinetic energy $\frac{1}{2}mv^2$ of the electron is equal in magnitude to the potential energy $\frac{\mathbf{Ze^2}}{\mathbf{4\pi\varepsilon_0 r}}$, then electron could escape from the electrostatic field of a nucleus.

$$\frac{1}{2}mv^2 = \frac{\mathbf{Ze^2}}{\mathbf{4\pi\varepsilon_0 r}} = nh\upsilon$$

$$\boxed{v = v_{escape} = \sqrt{\frac{2nh\upsilon}{m}}}$$

$$\frac{mv^2}{r} = \frac{Ze^2}{4\pi\varepsilon_0 r^2}$$

$$\boxed{m\omega^2 r = \frac{Ze^2}{4\pi\varepsilon_0 r^2}}$$

$$\left(\frac{2\pi}{T}\right)^2 = \frac{e^2}{4\pi\varepsilon_0 m} \times \frac{Z}{r^3}$$

$$\left(\frac{2\pi}{T}\right)^2 = r_e\, c^2 \times \frac{Z}{r^3}$$

where r_e denote the Classical electron radius

$$\boxed{T^2 \propto \frac{r^3}{Z}}$$

"The very nature of the quantum theory ... forces us to regard the space-time coordination and the claim of causality, the union of which characterizes the classical theories, as complementary but exclusive features of the description, symbolizing the idealization of observation and description, respectively."

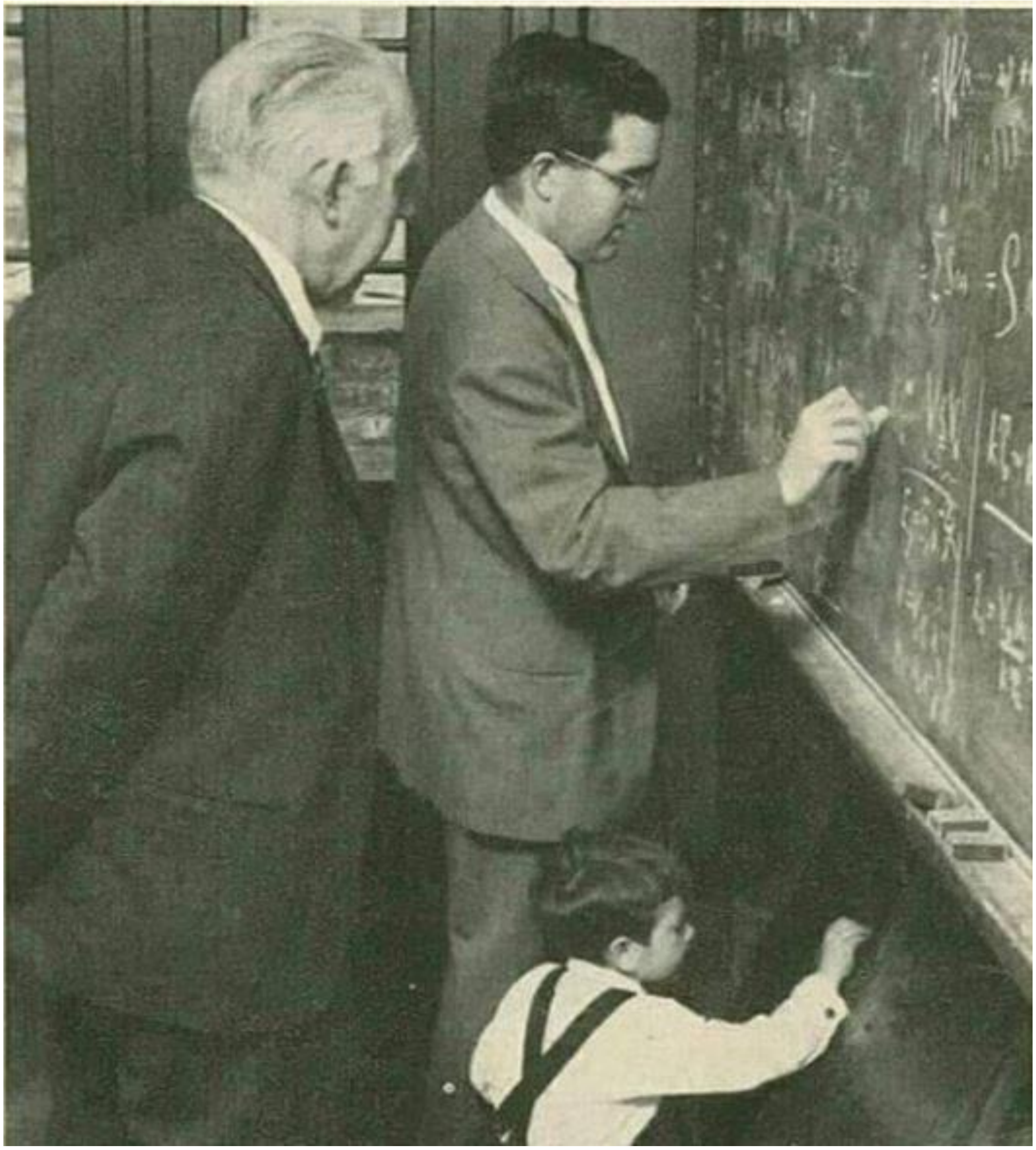
— Niels Bohr

Bohr and Margrethe Norlund on their engagement in 1910

1927 Solvay Conference in Brussels, October 1927. Bohr is on the right in the middle row, next to Max Born

Werner Heisenberg (left) with Bohr at the Copenhagen Conference in 1934

Bohr with James Franck, Albert Einstein and Isidor Isaac Rabi



Niels Bohr (left) – Danish theoretical physicist and winner of the 1922 Nobel Prize in Physics – along with his son Aage Bohr (center) – nuclear physicist and winner of the 1975 Nobel Prize in Physics – and his grandson Tomas Bohr (bottom)

Total energy of the electron:

$$E = -\frac{nh\upsilon}{2}$$

$$\boxed{\frac{E}{mc^2} = -\frac{n\upsilon}{2\upsilon_c}}$$

$\upsilon_c = \dfrac{mc^2}{h}$ is the Compton frequency of the electron.

Orbital velocity:

$$\frac{mv^2}{r} = \frac{Ze^2}{4\pi\varepsilon_0 r^2}$$

$$\boxed{v = \sqrt{\frac{Ze^2}{4\pi\varepsilon_0 rm}} = \sqrt{\frac{nh\upsilon}{m}}}$$

$$v = \sqrt{\frac{Ze^2}{4\pi\varepsilon_0 rm}} = c\sqrt{\frac{Z \times \text{Classical electron radius}}{r}}$$

The moment of inertia of an electron in n^{th} orbit is:

$$I = n \times mr^2$$

Since:

$$L = n\hbar$$

Therefore:

$$I = \frac{L}{\hbar} \times mr^2$$

$$\boxed{I = \frac{L^2 r}{\hbar v} = \frac{L^2}{\hbar \omega}}$$

The acceleration of the electron:

$$a = \frac{v^2}{r} = \omega \times v$$

$$\boxed{a = \frac{2\pi}{T} \sqrt{\frac{nh\upsilon}{m}}}$$

$$F = \frac{Ze^2}{4\pi\varepsilon_0 r^2}$$

Since:

$$\text{Fine structure constant } (\boldsymbol{\alpha}) = \frac{e^2}{4\pi\varepsilon_0 \hbar c}$$

Therefore:

$$\boxed{F = \boldsymbol{\alpha} \times Z \times \frac{\hbar c}{r^2} = \boldsymbol{\alpha} \times Z \times \frac{hc}{2 \times \text{Area of circular orbit}}}$$

Quantum circulation:

$$Q_0 = \frac{h}{2m}$$

Since:

$$mvr = \frac{nh}{2\pi}$$

Therefore:

$$v = \frac{nQ_0}{\pi r} = \frac{2Q_0}{\lambda}$$

$$\boxed{\omega = \frac{v}{r} = \frac{nQ_0}{\text{Area of circular orbit}}}$$

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